N11/5/MATHL/HP2/ENG/TZ0/XX

88117202

MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 3 November 2011 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

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13 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Consider the graph of $y = x + \sin(x - 3)$, $-\pi \le x \le \pi$.

(a) Sketch the graph, clearly labelling the x and y intercepts with their values. *[3 marks]*

(b) Find the area of the region bounded by the graph and the x and y axes. *[2 marks]*

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2. *[Maximum mark: 7]*

Given the following system of linear equations,

$$ax + y + z = 1$$
$$x + ay + z = a$$
$$x + y + az = a^2$$

find the values of the real constant, a, for which the system has a unique solution.

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3. *[Maximum mark: 5]*

The number of vehicles passing a particular junction can be modelled using the Poisson distribution. Vehicles pass the junction at an average rate of 300 per hour.

(a) Find the probability that no vehicles pass in a given minute. *[2 marks]*

(b) Find the expected number of vehicles which pass in a given two minute period. *[1 mark]*

(c) Find the probability that more than this expected number actually pass in a given two minute period. *[2 marks]*

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4. *[Maximum mark: 5]*

(a) Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a$, $a \in \mathbb{Q}^+$, find the value of a. *[3 marks]*

(b) Hence, or otherwise, solve the equation $\arcsin x = \arctan a$. *[2 marks]*

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5. *[Maximum mark: 5]*

The probability that the 08:00 train will be delayed on a work day (Monday to Friday) is $\frac{1}{10}$. Assuming that delays occur independently,

(a) find the probability that the 08:00 train is delayed exactly twice during any period of five work days; *[2 marks]*

(b) find the minimum number of work days for which the probability of the 08:00 train being delayed at least once exceeds 90 %. *[3 marks]*

6. *[Maximum mark: 7]*

The complex numbers z_1 and z_2 have arguments between 0 and π radians. Given that $z_1 z_2 = -\sqrt{3} + \mathrm{i}$ and $\frac{z_1}{z_2} = 2\mathrm{i}$, find the modulus and argument of z_1 and of z_2.

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7. *[Maximum mark: 6]*

(a) Find the set of values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2x}{x+1}\right)^n$ has a finite sum. *[4 marks]*

(b) Hence find the sum in terms of x. *[2 marks]*

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8. *[Maximum mark: 7]*

Given that $f(x) = \frac{1}{1+e^{-x}}$,

(a) find $f^{-1}(x)$, stating its domain; *[6 marks]*

(b) find the value of x such that $f(x) = f^{-1}(x)$. *[1 mark]*

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9. *[Maximum mark: 6]*

A stalactite has the shape of a circular cone. Its height is 200 mm and is increasing at a rate of 3 mm per century. Its base radius is 40 mm and is decreasing at a rate of 0.5 mm per century. Determine if its volume is increasing or decreasing, and the rate at which the volume is changing.

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10. *[Maximum mark: 7]*

Given that $z = \frac{2-\mathrm{i}}{1+\mathrm{i}} - \frac{6+8\mathrm{i}}{u+\mathrm{i}}$, find the values of u, $u \in \mathbb{R}$, such that $\operatorname{Re} z = \operatorname{Im} z$.

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 15]*

Jan and Sia have been selected to represent their country at an international discus throwing competition. Assume that the distance thrown by each athlete is normally distributed. The mean distance thrown by Jan in the past year was 60.33 metres with a standard deviation of 1.95 metres.

(a) In the past year, 80 % of Jan's throws have been longer than x metres. Find x correct to two decimal places. *[2 marks]*

(b) In the past year, 80 % of Sia's throws have been longer than 56.52 metres. If the mean distance of her throws was 59.39 metres, find the standard deviation of her throws. *[3 marks]*

(c) This year, Sia's throws have a mean of 59.50 metres and a standard deviation of 3.00 metres. The mean and standard deviation of Jan's throws have remained the same. In the competition, an athlete must have at least one throw of 65 metres or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round.

(i) Determine whether Jan or Sia is more likely to qualify for the final on their first throw.

(ii) Find the probability that both athletes qualify for the final. *[10 marks]*

12. *[Maximum mark: 16]*

(a) In an arithmetic sequence the first term is 8 and the common difference is $\frac{1}{4}$.
If the sum of the first $2n$ terms is equal to the sum of the next n terms, find n. *[9 marks]*

(b) If $a_1, a_2, a_3, \ldots$ are terms of a geometric sequence with common ratio $r \neq 1$, show that $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + \ldots + (a_n - a_{n+1})^2 = \frac{a_1^2(1-r)(1-r^{2n})}{1+r}$. *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

13. *[Maximum mark: 16]*

Two planes Π_1 and Π_2 have equations $2x + y + z = 1$ and $3x + y - z = 2$ respectively.

(a) Find the vector equation of L, the line of intersection of Π_1 and Π_2. *[6 marks]*

(b) Show that the plane Π_3 which is perpendicular to Π_1 and contains L, has equation $x - 2z = 1$. *[4 marks]*

(c) The point P has coordinates $(-2, 4, 1)$, the point Q lies on Π_3 and PQ is perpendicular to Π_2. Find the coordinates of Q. *[6 marks]*

14. *[Maximum mark: 13]*

(a) Show that $\left| e^{i\theta} \right| = 1$. *[1 mark]*

Consider the geometric series $1 + \frac{1}{3}e^{i\theta} + \frac{1}{9}e^{2i\theta} + \dots$.

(b) Write down the common ratio, z, of the series, and show that $|z| = \frac{1}{3}$. *[2 marks]*

(c) Find an expression for the sum to infinity of this series. *[2 marks]*

(d) Hence, show that $\sin\theta + \frac{1}{3}\sin 2\theta + \frac{1}{9}\sin 3\theta + \dots = \frac{9\sin\theta}{10 - 6\cos\theta}$. *[8 marks]*